Hybrid Manual Errata, First Printing

Chapter 3, page 21- equation 1 should read G = wdoto/Ac

page 22- fourth paragraph “”The “holy grail” of regression rates was 0.1 inch/second from (for) some time….

In chapter 4 page 25 second to last paragraph first sentence. "Try the core diameter which is limited by G: First how much oxidizer if (is) flowing?"

page 27 where the math is incorrect. For that entire section substitute the following:

> ” Lets start with a system that requires 5000 lbf thrust for 15 seconds. The only envelope restriction is a diameter of 10 inches.
>
> Lets assume the following properties for the propellant combination of N_2O and HydroxyTerminated PolyButadiene-
>
> 1. Oxidizer density 48 lbm/ft3
> 2. Fuel density of .032 lbm/in3
> 3. O/F ratio of 4.0 (off Stoichiometric)
> 4. Operating pressure of 350 psi
> 5. Isp of 200 sec
> 6. “a” = .03
> 7. “n” = 0.5
>
> By choosing a fuel-rich O/F ratio, we lower the combustion temperature in this case, allowing the use of a simple phenolic nozzle. The Isp lowers only slightly.
>
> Try the core diameter which is limited by G: First how much oxidizer if flowing? That is determined by dividing thrust by Isp: 5000/200 = 25 lbm/sec propellants. For 4.0:1 O/F to get oxidizer flow rate: [25/(4.0+1)]*4 = 20 lbm/sec. Using a maximum G value of 0.8 lbm/in2/sec, the area of the core is: 20/.8 = 25 in^2. The core diameter then is: (25/.7854)^.5 = 5.64 inches. Note that this core does not have to be circular, but it does have to have this area as the absolute minimum.

Now let us look at a parameter not discussed above, the ratio of the core area to the area of the nozzle throat. This is set to be above 2.0 to reduce effects of high velocity flow in the core. We will have to look up a value for C_f for our propellant combination and at the specified pressure. That is most easily done with a chemical equilibrium program such as PROPEP, freely available on the Internet. There are three GUI's for PROPEP of interest, the Gas Dynamics Laboratory's GDLPROPEP, wpropepz, and the Art Lekstutis' GUIPEP. All are recommended for quick looks at propellant combinations. "ISP" is the USAF dos variant.

Our C_f might come in at a value of 1.4 for this pressure and propellant combination. So the throat area is: 5000/(350 *1.4) = 10.2 square inches. The core/throat area ratio then is: 25/10.2 = 2.45 or a bit above the 2.0 recommended.

So we can proceed with the design. We will increase the core diameter a bit to 6 inches since that will give us a bit of margin on the critical value of G, increase the ratio to the nozzle throat a bit more and move us towards using a possible standard size material for a mandrel.

How much fuel is being consumed during the burn? The total is 25 lbm/sec of which 20 lbm/sec is oxidizer. That leaves us with 5 lbm/sec for fuel consumption rate. What is that in terms of volume? 5/.032 = 156.3 in^3 is the volume consumed per second. If we determine the regression rate, we can calculate the length of the grain.

r_b = a G^n. Lets say our propellant has an a value of .03 and an n value of 0.5. Our new G value is 20/(6*6*.7854) = .71. So our r_b = .03 * .71^.5 = .025 in/sec.

The area of the core then is 156.3/.025 = 6120 in^2. The core perimeter is 6 * 3.1416 = 18.84 inch. So the length of the grain is 6120/18.84 = 324 inches. Whoops! What has happened? We have a grain that is nearly 30 feet long! Clearly our outcome does not fit the real world of possibilities.

Note that a higher regression rate will reduce the length linearly so the holy grail of 0.1 inch/sec mentioned previously would bring the length down to 81 inches. Since we have a diameter of 10 inches to use, we are greatly remiss in not making the core larger. The attraction of a high G moved us to use the smallest possible core. Note that at .025 in/sec, at 15 seconds we have only used a web thickness of .375 inch. That makes our overall design perhaps 7.25 inch including insulation and wall thickness of the case.

So do we have a higher regression rate? If not, we will increase the diameter of the core. The second way of approaching the motor design is to start with the envelope diameter. Using 10 inches as a diameter, we can pick a core diameter estimated on an assumed r_b, or perform a set of converging calculations which will give us the core diameter based on the equation for G and the resultant r_b.

Lets see what happens if we had chosen an 8.5 inch core (I picked that intuitively in this example). The G value is: 20/(8.5*8.5*.7854) = .352.

The regression rate then is: .03 * .352^.5 = .018 in/sec.

The web thickness for 15 seconds of burn then is:

15 * .018 = .27 inch. The diameter of the grain then is: 8.5 + 2*.27 = 9.04 inch. This gives plenty of thickness for an insulator or cartridge case and the motor wall thickness. If we did a convergence we would end up with a slightly larger core diameter.

With this new core diameter, 8.5 inches, what is the overall length of the grain? The area now of burn at the new rate is: 156.3/.018 = 8683 $inch^2$. The perimeter of the core is: 8.5 * 3.1416 = 26.7 inch. The length then is: 8683/26.7 = 325 inch. Now that is nearly 30 feet in length. What choices do we have here? If we configure the grain with a complex geometry, we can greatly reduce the length. Imagine placing 14 each, .54 inch by 2.16-inch rectangular rods bonded to the inner surface of the core. This makes an inverted gear looking shape. They would have a spacing of about .5 inch. They take up .54 inch of the surface but add 4.32 inches of linear surface. This technique will triple the area reducing our grain length now to about 9 feet in length. Other more complex shapes can reduce it even more. Now we are not going to glue bars into our grain, we will make a mandrel that creates this shape (although we have designed grains of acrylic in which we adhesively bonded acrylic bars to the interior of tubes to make very nice grains indeed). In addition, we might add some additional web to join the inner ends of the bars for support during burn.

Note that this motor is very inefficient in its use of volume. This is why higher regression rates are being sought.

We now have a motor 10 inches in diameter with an inverted gear shaped core inner diameter is 4.6 inches and larger diameter is 8.5 inches. We have just reduced the core area, so now our G has gone up a bit to about 0.5 and the new regression rate should be calculated. The core to throat area ratio is about 4.0, a very high value. We actually had a lot of additional web thickness in terms of HTPB being used for insulation, so we can forgo a redesign, we are just ending up with a motor which will have a large sliver fraction at the end of burn. That is to say much of the fuel portion will be unburned. We can still proceed to static test, however, with a motor that is guaranteed to burn for 15 seconds with no instance of case burn-through.

Since we are using a self-pressurizing oxidizer, the tank design is simple. The total weight of oxidizer is: 20*15 = 300 lbm. But we know that about 10% of the oxidizer will be gas at the end of burn and really not contribute to the total impulse. So we add that amount initially. The volume of the oxidizer is: (300+30)/48 = 6.88 $feet^3$ or: 1728 * 6.88 = 11888 $inch^3$. For a 10 inch tank with a .25 inch wall (we will check that wall thickness later) the length is: 11888/(9.5*9.5*.7854)= 168 inch. We will add a few inches to gain about a 5% minimum ullage. The overall length of our propulsion system is about 23 feet. At 10 inches in diameter it has a somewhat high fineness ratio.

This is the basic dimensioning of the two larger elements of the propulsion system, the motor case (containing the grain) and the oxidizer tank. In the next section we will move on to other areas needed to complete the design."

Clarifications:

from page 25 paragraph 6

1) O/F ratio of 4.0:1   This ratio is used as an example. For N2O HTPB a higher Isp value would be 6.5:1 for instance.2) To get oxidizer flow rate: (25/5)*4=20 lbm/sec, the value "5" is the O/F ratio + 1

3) .8 is used for max G value, this value is determined experimentally and is the upper limit for most combinations of N2O + Fuel.

4) The core diameter then is: (25/.7854)^.5=5.64 inches, the value ".7854" is PI/4

In the last paragraph on page 25

5) Core area to nozzle area is set above 2.0:1; this is a value that will assure that high velocity conditions do not exist in the core.

On page 26 second paragraph

6) Cf is always taken at the chamber pressure/exit pressure ratio

last paragraph 5th sentence

7) The diameter of the grain then is 8.5 + 2*.27=9.04 inch, the "2" is the multiplier for a single web thickness.